

September 2014

The main program is anQCD.m. It is written in Mathematica, and is checked to work in Mathematica 9.0.1 and in Mathematica 10.0.1. It is believed that the program works also on several earlier versions of Mathematica.

In order to call the main program, one should first open Mathematica window, with the command: mathematica, and then clicking on: Notebook (in Mathematica 9); clicking on: New Document (in Mathematica 10). The Mathematica window should be opened in the archive where anQCD.m and the auxiliary subprograms Li__nu.m and s0r.m are situated on your computer.

After opening the Mathematica window, the program is then called with the command:

```
<<anQCD.m
```

The program anQCD.m automatically calls also the (sub)programs Li__nu.m and s0r.m. It needs about 10 seconds to load.

The main functions are the analytic (holomorphic) couplings $A_{\nu}(Q^2)$ for three different analytic QCD models: FAPT (and FAPTglobal); 2delta anQCD [2d]; massive PT [MPT].

These couplings are explained at the beginning of the text of the file (program) anQCD.m.

Some examples of these explanations:

AFAPT3l::usage = "AFAPT3l[Nf, nu, k, Q2, L2, Fi] gives the 3-loop FAPT coupling with the inclusion of logarithm of the coupling in the form $[a3l]^{\nu} \text{Log}[a3l]^k$ $_{\{an.FAPT\}}$, where [...] $_{\{an.FAPT\}}$ is the FAPT-analytization of the term in [...]; Nf is the number of active flavors; nu real index; k=0,1,2,...; $Q2=|Q^2|$; $L2=\{\bar{\Lambda}\}^2_{\{Nf\}}$ (both Q2 and L2 in GeV^2); $Fi=\text{Arg}[Q^2]$, i.e., $Q^2 = Q2 \text{Exp}[I*Fi]$.";

AFAPT2lglob::usage = "AFAPT2lglob[nu, k, Q2, L2nf3, fi] gives the 2-loop global FAPT coupling with the inclusion of logarithm of the coupling in the form $[a2lglob]^{\nu} \text{Log}[a2lglob]^k$ $_{\{an.FAPT\}}$, where [...] $_{\{an.FAPT\}}$ is the FAPT-analytization of the term in [...]; L2nf3 is the (2-loop) Lambda scale (in GeV^2) at Nf=3 active quark flavors; other notations as in AFAPT1l.";

tA2d::usage = "tA2d[Nf, nu, Q2, Fi] gives the 2danQCD tilde coupling $\{\tilde{A}\}_{\nu}(Q^2;Nf)$ in the c2-Lambert scheme [the underlying coupling is a2d of Eq. (14) with $c2=c22din$] at $Q^2=|Q^2|\text{Exp}[I Fi]$, with $Q2=|Q^2|$ (in GeV^2), and Nf=3,4,5,6 is the number of active quark flavors.";

A2d5l::usage = "A2d5l[Nf, n, nu, Q2, Fi] gives the N=5 ('5-loop') 2danQCD coupling $A_{\{\nu+n\}}(Q^2;Nf)$, the 2danQCD analog of the pQCD power $a(Q^2;Nf)^{\{\nu+n\}}$; $Q2=|Q^2|$ in GeV^2 ; $Q^2=Q2\text{Exp}[I Fi]$; nu is any real index ($-1 < \nu$), and n is an integer: n=0,1,2,3,4; $A_{\{\nu+n\}}$ is constructed from $\{\tilde{A}\}_{\{\nu+n+m\}}$'s using Eq.(28) of the manuscript with N=5 (and $\nu=\nu_0$):
 $A2d3l[Nf,n,nu,Q2,Fi]=tA2d[Nf,nu+n,Q2,Fi]+ \text{Sum}(tk_m(\nu+n)*tA2d[Nf,nu+n+m,Q2,Fi])$
where the Sum runs from m=1 to m=N-1-n (where: N=5)."

tAMPT3l::usage = "tAMPT3l[Nf, n, del, Q2, M2, L2]= $\{\tilde{A}\}_{\nu=n+del}(Q^2=Q2)$ in 3-loop MPT with $m_{\{gl\}}^2=M2$ (in GeV^2), and $M\bar{S} \Lambda_{\{Nf\}}^2=L2$; where Q2 is COMPLEX $Q2=Q^2=|Q^2| \text{Exp}(I fi)$ (in GeV^2); $\nu=n+del$ is the index of generalized logarithmic derivative, where n=0,1,3,4; $0=del<1$."

AMPT4l::usage = "AMPT4l[Nf,Nu,Q2,M2,L2] gives the N=4 (4-loop) MPT coupling $A_{\nu}(Q^2;N_f)$, the MPT analog of the pQCD power $a(Q^2;N_f)^{\nu}$; $Q^2=Q^2$ in GeV^2 (complex number in general); M^2 is the MPT scale (in GeV^2); $0 < \nu < 5$; A_{ν} is constructed from $\{\tilde{A}_{\nu+m}\}$'s using Eq.(28) with truncations as explained in the text at the end of Subsection 3.4 of the manuscript.";

In Mathematica window (Notebook), these explanations can be called by typing the command ?nameoffunction; for example:

```
?A2d2l
```

More detailed explanations are given in the text of the manuscript (article), especially Section 4 and Appendix A.

Some examples:

In the examples below, the timing option is included, the time of calculation is the first entry in the output and it varies with various computers and with various runs; furthermore, the time is somewhat (20-50%) longer in Mathematica 10.0.1. than in Mathematica 9.0.1:

```
In[1]:= <<anQCD.m;
(* The program needs about 10-15 seconds to load *)
*-----*)
(* First with Nf=3; nu=1; Q^2=10.^-3 GeV^2; barLambda^2(Nf=3)=0.1 GeV^2): *)
(* Note that the result is the second entry; the first entry is the time (in
seconds, using Mathematica 9.0.1 on a laptop, with Linux OS): *)
```

```
In[2]:= AFAPT3l[3, 1, 0, 10.^-3, 0.1, 0] // Timing
Out[2]= {0.399939, 0.28312}
```

```
(* In FAPT global no Nf needed: *)
In[3]:= AFAPT3lglob[1, 0, 10.^-3, 0.1, 0] // Timing
Out[3]= {0.829874, 0.287775}
```

```
(* In 2-delta anQCD: *)
In[4]:= A2d3l[3, 0, 1, 10.^-3, 0] // Timing
Out[4]= {0.419936, 0.809041}
```

```
(* In MPT we choose here: m^2_{gl}=0.7 GeV^2: *)
In[5]:= AMPT3l[3, 1, 10.^-3, 0.7, 0.1] // Timing
Out[5]= {0.142978, 0.171356}
```

```
(*-----*)
(* Now the same, but at higher momentum squared: Q^2=10^2 GeV; and Nf=5: *)
```

```
In[6]:= AFAPT3l[5, 1, 0, 10^2, 0.1, 0] // Timing
Out[6]= {0.414936, 0.0624843}
```

```
In[7]:= AFAPT3lglob[1, 0, 10^2, 0.1, 0] // Timing
Out[7]= {0.821874, 0.0559854}
```

```
In[8]:= A2d3l[5, 0, 1, 10^2, 0] // Timing
Out[8]= {0.514922, 0.0559197}
```

```
In[9]:= AMPT3l[5, 1, 10^2, 0.7, 0.1] // Timing
Out[9]= {0.111983, 0.0627726}
```

```
(*-----*)
(* Now various couplings at index nu=1.7 instead of nu=1 and at complex  $Q^2 = 0.6 \text{ Exp}[i \ 0.9]$  (and:  $N_f=3$ ;  $\bar{\Lambda}^2(N_f=3)=0.1 \text{ GeV}^2$ ): *)
```

```
In[10]:= AFAPT3l[3, 1.7, 0, 0.6, 0.1, 0.9] // Timing
Out[10]= {0.387942, 0.0204301 - 0.00398595 I}
```

```
In[11]:= AFAPT3lglob[1.7, 0, 0.6, 0.1, 0.9] // Timing
Out[11]= {0.879867, 0.0216249 - 0.00399341 I}
```

```
In[12]:= tA2d[3, 1.7, 0.6, 0.9] // Timing
Out[12]= {0.753886, 0.0358565 - 0.0228659 I}
```

```
In[13]:= A2d3l[3, 1, 0.7, 0.6, 0.9] // Timing
Out[13]= {1.494773, 0.0294109 - 0.0172694 I}
```

```
In[14]:= tAMPT3l[3, 1, 0.7, 0.6 Exp[I 0.9], 0.7, 0.1] // Timing
Out[14]= {0.053992, 0.0262399 - 0.00147327 I}
```

```
In[15]:= AMPT3l[3, 1.7, 0.6 Exp[I 0.9], 0.7, 0.1] // Timing
Out[15]= {0.186971, 0.0249888 - 0.00286272 I}
```

```
(*-----*)
```